Journal of Structural Geology 32 (2010) 685–692

Contents lists available at ScienceDirect

Journal of Structural Geology

journal homepage: www.elsevier.com/locate/jsg

A rapid Bézier curve method for shape analysis and point representation of asymmetric folds

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ARTICLE INFO

Article history:
Received 5 August 2009
Received in revised form
30 March 2010
Accepted 9 April 2010
Available online 24 April 2010

Keywords:
Asymmetric fold
Bézier curve
Inflection point
Lift
Point representation

ABSTRACT

Point representation of fold shapes is useful, in particular, for classification of a large number of folds into different geometric populations. The methods for shape analysis and point representation of asymmetric folds are a few and tedious, although several methods exist for the analysis of the individual fold limbs, or symmetric folds. This article gives a rapid method that uses the Bézier curve tool, available in any common computer graphics software, for the analysis of a complete asymmetric fold and its point representation in the two-dimensional frame.

The new method is based on the reduction of variables in the parametric equations of a cubic Bézier curve. It makes the length of one Bézier handle zero, pins the end point of the other Bézier handle at the origin of the X–Y frame and drags its control point along the Y-axis to fit the Bézier curve on the given asymmetric fold. A Cartesian plot between normalised length of the Bézier handle and the lift, i.e., difference between the heights of the two inflection points, gives the unique point that represents the given asymmetric fold shape. We test the validity of the new method on several computer simulated asymmetric folds and demonstrate its usefulness with the help of a natural example.

1. Introduction

Structural geologists commonly require distinctions between the shapes of minor folds that may occur in different parts of a large scale fold or belong to one or more groups/domains. Although any scheme that displays a large number of fold shapes on a single plot is useful for classification of folds into different shape populations, the point representation in the two-dimensional Cartesian frame is ideal. This article proposes a Bézier curve method for the analysis and point representation of asymmetric fold shapes. The new method, an extension of the Bézier curve method for analysis of individual fold limbs (Srivastava and Lisle, 2004), is rapid and easier than existing methods for the fold shape analysis and the point representation (Tripathi and Gairola, 1999; Bastida et al., 2005). All the fold shapes referred in this article are the profile sections of single folded surfaces, e.g., contact surface between the adjacent beds/lithological layers.

2. Geometrical attributes of asymmetric folds

2.1. Fold shapes

Loudon (1964) and Whitten (1966, pp. 586–590) define fold shapes by the statistical attributes, the fourth statistical moment, or Kurtosis. The interlimb angle or tightness, and the attributes such as dextral (z-shaped) or sinistral (s-shaped) shapes have long been used for description of asymmetric fold shapes (Fleuty, 1964; Ramsay, 1967, pp. 351–354). As two or more folds of equal interlimb angle can have different shapes, additional attributes are necessary for their complete description (Ramsay, 1967, pp. 350–351). In the most comprehensive account till date, Twiss (1988) shows that a complete description of an asymmetric fold shape requires, at least, six parameters: the aspect ratio, the folding angle (180° − interlimb angle), the bluntness of the closure, the two inclination angles and the hinge tangent angle.

Existing methods for the shape analysis of individual limbs or symmetric folds include the harmonic analysis (Chapple, 1968; Stabler, 1968; Hudleston, 1973; Ramsay and Huber, 1987, p. 314; Stowe, 1988), the conic section analysis (Aller et al., 2004), the power function analysis (Bastida et al., 1999), the super ellipses (Lisle, 1988) or the Bézier curve analysis (Srivastava and Lisle, 2004;

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doi:10.1016/j.jsg.2010.04.002

Lisle et al., 2006). The results of limb-by-limb analysis can represent an asymmetric fold by a tie-line that joins the two points on a Cartesian plot, one corresponding to each fold limb (Fig. 15.14 in Ramsay and Huber, 1987, p. 317; Hudleston, 1973). Distinction between different populations of the asymmetric fold shapes is, however, difficult on the plots that contain a large number of intersecting tie-lines.

2.2. Degree of asymmetry

The degree of asymmetry, an important geometrical attribute of asymmetric folds, has been used for interpretations regarding the large scale fold geometry, and the buckling/bending moment ratio during the process of buckle folding (Price, 1967; Price and Cosgrove, 1990, p. 328). Loudon (1964) and Whitten (1966, p. 588) define degree of asymmetry by the third statistical moment or skewness. Several other definitions of the degree of asymmetry exist. For example, Price (1967) defines the degree of asymmetry as the limb length ratio. This definition, however, does not account for the fold shape, because two asymmetric folds with the same limb length ratio may have different shapes. Similarly, the degree of asymmetry, expressed as the angle between bisector of the folding angle (180° − interlimb angle) and the median trace (Twiss and Moores, 1992, p. 207) also does not give any information about the fold shape.

In yet another definition, the degree of asymmetry is the sum of the differences in the aspect ratio Δ_{size} and, the shape Δ_{shape} of the two limbs (Tripathi and Gairola, 1999). The scope of this definition is, however, limited because several combinations of the two differences, Δ_{size} and Δ_{shape}, may yield the same degree of asymmetry. Bastida et al. (2005) improve this scheme by defining the degree of asymmetry as the ratio of 'shape asymmetry' and 'size asymmetry' and show that an asymmetric fold shape can be represented as a point on the two-dimensional plot of the two types of asymmetry. They define the 'shape asymmetry' as the ratio of the normalised areas of the opposite limbs, where the normalised area itself is the ratio of limb area and rectangular area that bounds the limb, and the 'size asymmetry' as the amplitude ratio of the two limbs. We propose a simple and rapid alternative method that traces the asymmetric fold shape by a Bézier curve and represents it as a point in the two-dimensional frame.

3. The Bézier curve method

Bézier curves are named after Bézier (1966), who used them for designing curvatures in the automobile industry. De Paor (1996) first demonstrated the potential of Bézier curves in structural geological applications, such as section construction and balancing, modeling fault displacement problems and representation of strain variations in thrust sheets and ductile shear zones. It is now well established that the Bézier curve tool, available in most computer graphics software, is a powerful tool for rapid and accurate analysis of fold shapes (Wojtal and Hughes, 2001; Srivastava and Lisle, 2004; Coelho et al., 2005; Lisle et al., 2006; Liu et al., 2009a,b).

3.1. Rationale

A Bézier curve consists of one or more polynomial segments such that each segment is defined by the following parametric equations (Davies et al., 1986; De Paor, 1996; Srivastava and Lisle, 2004).

$$x(t) = (1-t)^3 x_0 + 3(1-t)^2 t x_1 + 3(1-t)t^2 x_2 + t^3 x_3 \tag{1}$$

$$y(t) = (1-t)^3 y_0 + 3(1-t)^2 t y_1 + 3(1-t)t^2 y_2 + t^3 y_3 \tag{2}$$

These equations are derived from the coordinates of the four points: two end points P_0 (x_0, y_0) and P_3 (x_3, y_3), and two control points, P_1 (x_1, y_1), P_2 (x_2, y_2) of the two Bézier handles, P_0P_1 and P_2P_3, respectively. Parameter t varies from 0 to 1 from the first end point P_0 to the second end point P_3, and it satisfies $0 < t < 1$ at all other points on the curve (Fig. 1a).

The shape of Bézier curve depends on eight variables, i.e., the coordinates of the four points, P_0, P_1, P_2, and P_3 (Fig. 1a). Such a curve cannot be directly represented as a point on the two-dimensional X–Y plane, because the number of variables is too large. The number of variables can, however, be reduced by defining a cubic Bézier curve in a X–Y Cartesian frame such that: (i) first Bézier handle, P_0P_1 lies on the Y-axis with its end point P_0 and the control point P_1 at (*0*, *0*) and (*0*, *c*), respectively, (ii) the control point P_2 of second Bézier handle, P_2P_3 coincides with its end point P_3 *(a, b)*. The length of second Bézier handle P_2P_3 is, therefore, zero (Fig. 1b). With these constraints, the shape of Bézier curve

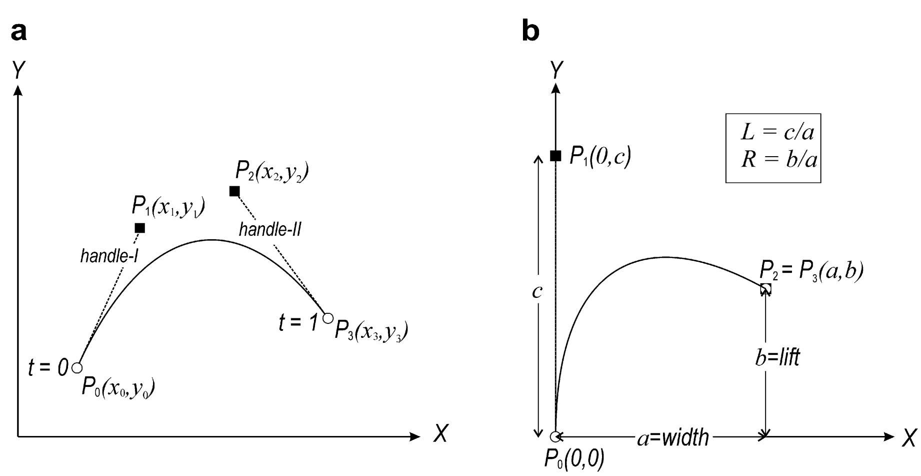

Fig. 1. (a) A cubic Bézier curve in the X–Y Cartesian frame. Eight variables, namely, the coordinates of four points P_0 (x_0, y_0), P_1 (x_1, y_1), P_2 (x_2, y_2) and P_3 (x_3, y_3) define the curve. Parameter t varies from 0 at P_0 to 1 at P_3. P_0 and P_3 – End points, P_1 and P_2 – Control points. P_0P_1 and P_2P_3 are the two Bézier handles. (b) Reduction in number of variables. P_0 is pinned at the origin (0, 0) and P_1 is moved such that the Bézier handle P_0P_1 lies on the Y-axis. P_2 is dragged to P_3 (a, b). The horizontal and the vertical separations between the points P_0 and P_3 represent the width a and the lift b of the fold, respectively. c is the optimum length of the Bézier handle P_0P_1 that is required for simulation of the curve in (a).

depends on only two parameters: (i) L, the ratio of length c of the first Bézier handle and width a of the fold, $L = c/a$, and (ii) R, the ratio of lift b and width a of the fold, $R = b/a$. As the procedure and results of the method are independent of the absolute values of a, b and c, we assume a to be of unit length. This simplification makes $L = c$ and $R = b$. The Bézier curve can now be described by the following parametric equations:

$$x(t) = 3(1-t)t^2 + t^3 \tag{3}$$

$$y(t) = 3L(1-t)^2 t + 3R(1-t)t^2 + t^3 R; \quad 0 < t < 1 \tag{4}$$

For given values of L and R, equations (3) and (4) yield a unique Bézier curve.

3.2. Procedure

The method requires: (i) a digital image of the profile section of an asymmetric fold, and (ii) a personal computer with any graphics software, such as CorelDraw, Adobe *Illustrator* or SmartDraw. Step-by-step procedure of the shape analysis of an asymmetric fold trace is as follows:

(i) Import the image of the given asymmetric fold into any Graphics software that has the Bézier curve tool. Mark the inflection points, P_0 and P_3 on the long and the short limb of the fold, respectively, and draw the tangent T at the point P_0 (Fig. 2a). Group all the objects, namely, the points P_0 and P_3, the tangent T and the given fold image.

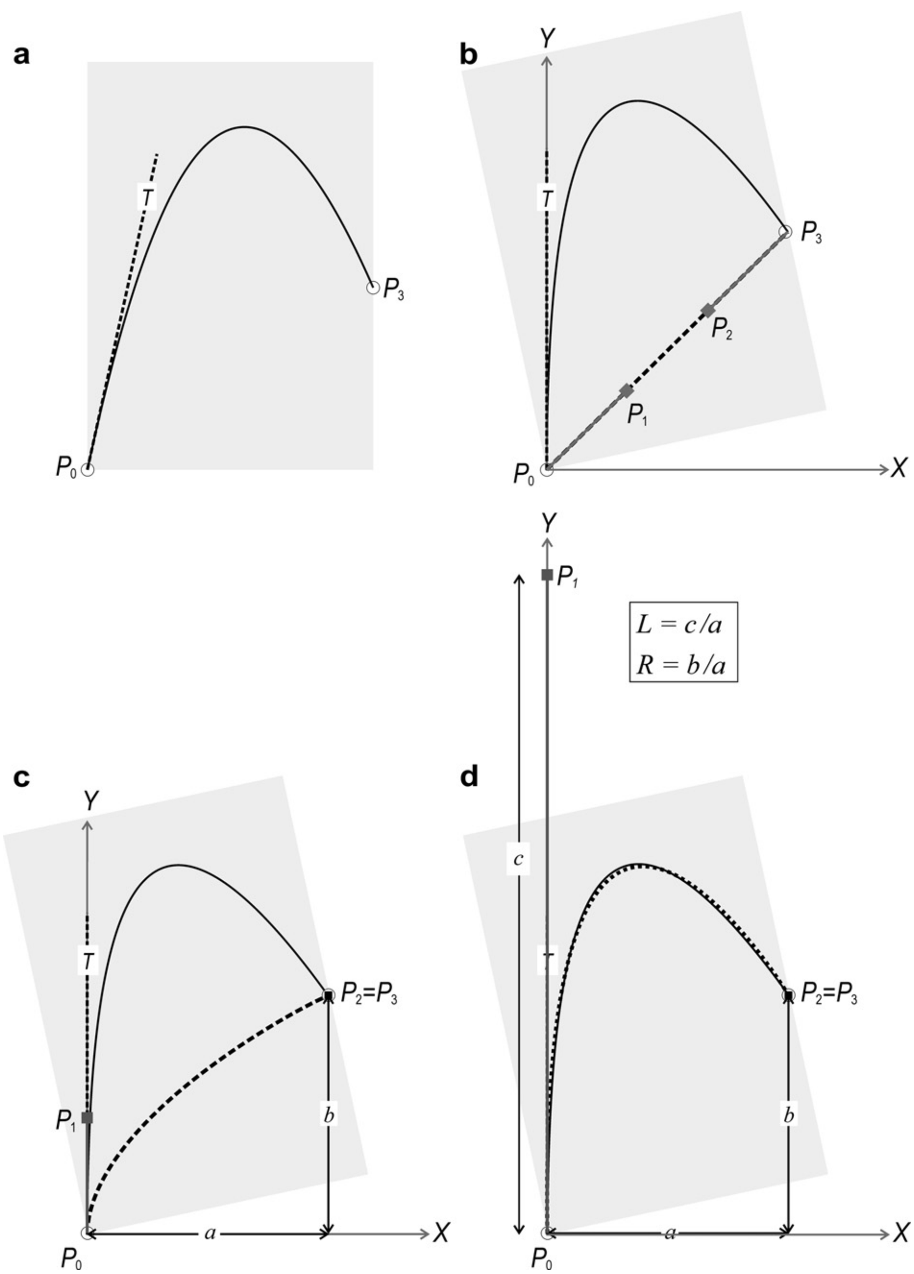

Fig. 2. Step-wise procedure for application of the Bézier curve method. (a) Imported image of the given asymmetric fold. P_0 and P_3 are the inflection points. T – Tangent at the inflection point P_0. (b) Rotation of the fold to make the tangent T vertical. Conversion of the P_0P_3 line (dashed) to the curve mode displays two Bézier handles P_0P_1 and P_2P_3. (c) P_2 is dragged to P_3, and P_1 is moved until it lies on the Y-axis. (d) P_1 is dragged along the Y-axis until the Bézier curve between the points P_0 and P_3 fits the given fold. a and b are the horizontal and vertical separations between the points P_0 and P_3, and c is the ordinate of point P_1.

(ii) Rotate the grouped objects in Fig. 2a such that the tangent T lies on the vertically directed Y-axis (Fig. 2b).

Join the inflection points P_0 and P_3 by a straight line and convert the line into curve mode by using the 'to curve' tool in the graphics softwares. This step displays two Bézier handles, P_0P_1 and P_2P_3, respectively (Fig. 2b).

(iii) Drag the control point P_2 such that it coincides with the end point P_3, and the length of the Bézier handle P_2P_3 becomes zero (Fig. 2c). Let the coordinates of the point $P_2 = P_3$ be (a, b), where a (width) and b (lift) are the horizontal separation and the vertical separation between the two inflection points, respectively (Fig. 2c).
(iv) While keeping the end point P_0 fixed, move the control point P_1 until the Bézier handle P_0P_1 lies on the Y-axis (Fig. 2c). This operation produces a Bézier curve (dashed curve in Fig. 2c).
(v) Drag the control point P_1 along the Y-axis until the Bézier curve, produced in the step-iv fits the given fold satisfactorily (Fig. 2d). The length of the Bézier handle, $P_0P_1 = c$ controls the shape of the given asymmetric fold (Fig. 2d).
(vi) Determine the parameter L by dividing the length c of Bézier handle P_0P_1 by a (Fig. 2d). Similarly, the second parameter R is the ratio b/a (Fig. 2d).

The Cartesian plot of the two parameters L and R represents the shape of the given asymmetric fold as a point.

4. Point representation

The validity of Bézier curve method has been tested on a large number of computer simulated and natural folds. We elucidate application of the method with the help of a few examples as follows:

4.1. Computer simulated folds

We have simulated asymmetric fold shapes by rotation and distortion of sine curve $y = A\sin x$ and parabola $y = -4Ax^2$ (Fig. 3a,b). These two curves are successively rotated by angles $\theta = 10°$, $20°$, $30°$, $40°$, $50°$ and $60°$ with respect to the horizontal reference line. Each rotated curve is distorted successively by the

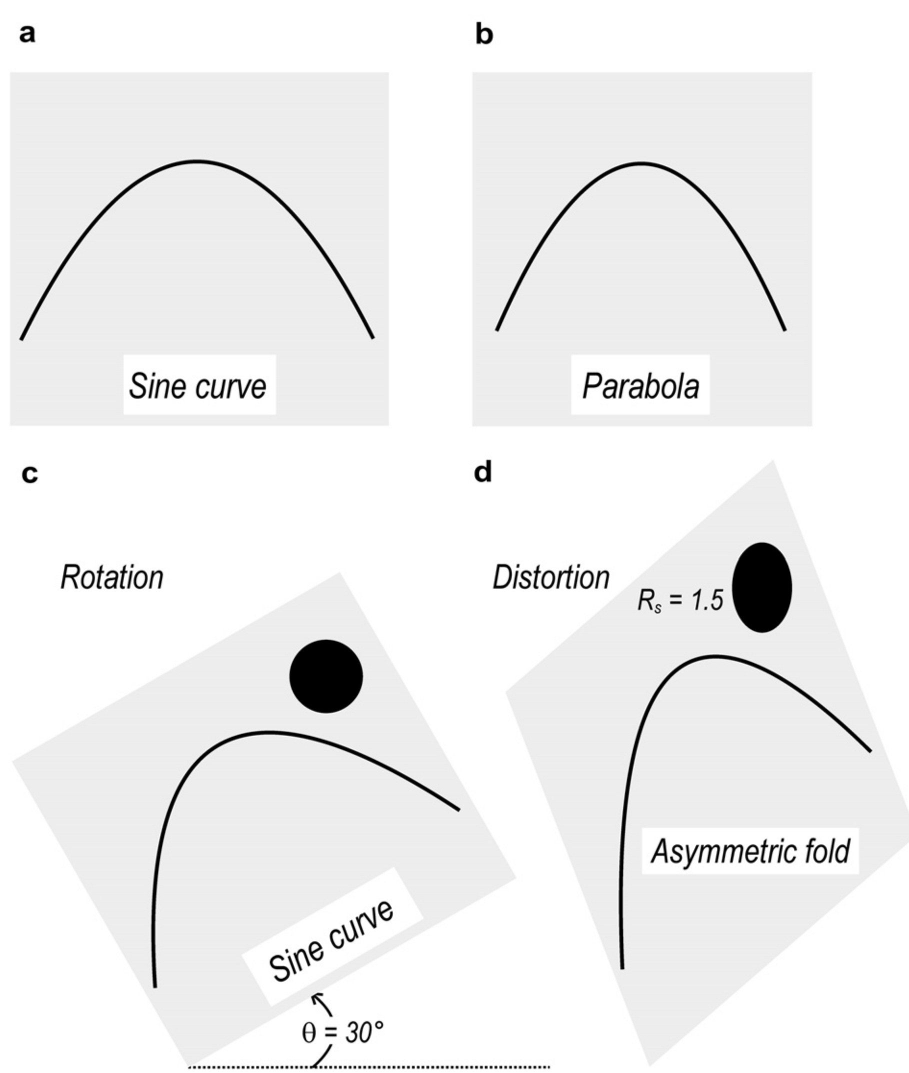

Fig. 3. Computer graphics simulation of the asymmetric fold shapes. (a) and (b) The sine curve and the parabola that are used for simulation of asymmetric fold shapes. (c) Rotation of the symmetric fold, the sine curve in (a) by an angle, $\theta = 30°$ in anticlockwise sense. Reference circle in black. (d) Distortion of image in (c) by the two-dimensional strain ratio $R_s = 1.5$ produces the asymmetric fold shape.

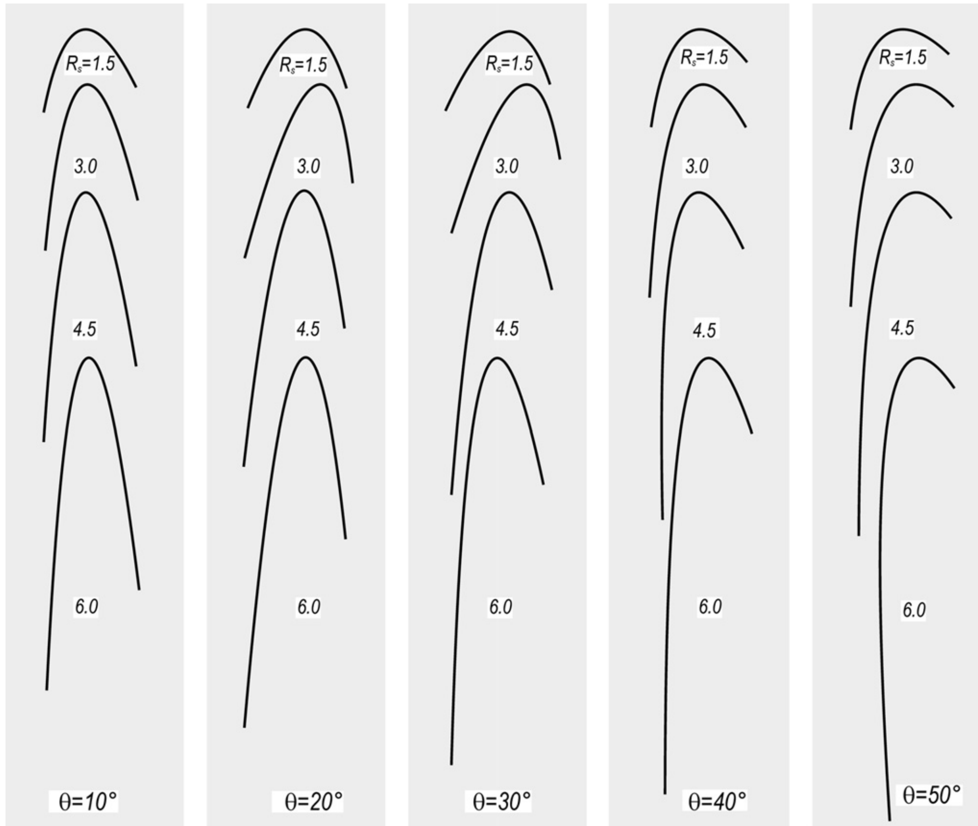

Fig. 4. Some examples of asymmetric fold shapes simulated from different combinations of rotation (θ) and distortion (R_s) of the sine curve in Fig. 3a.

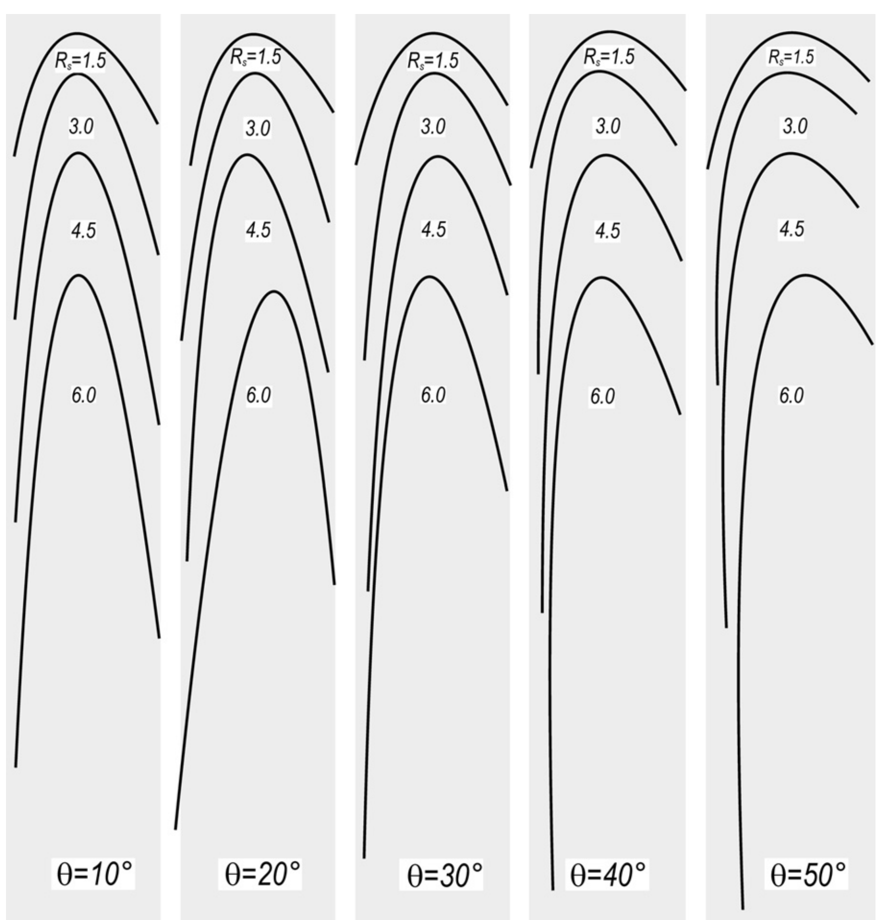

Fig. 5. Some examples of asymmetric fold shapes simulated from different combinations of rotation (θ) and distortion (R_s) of the parabola in Fig. 3b.

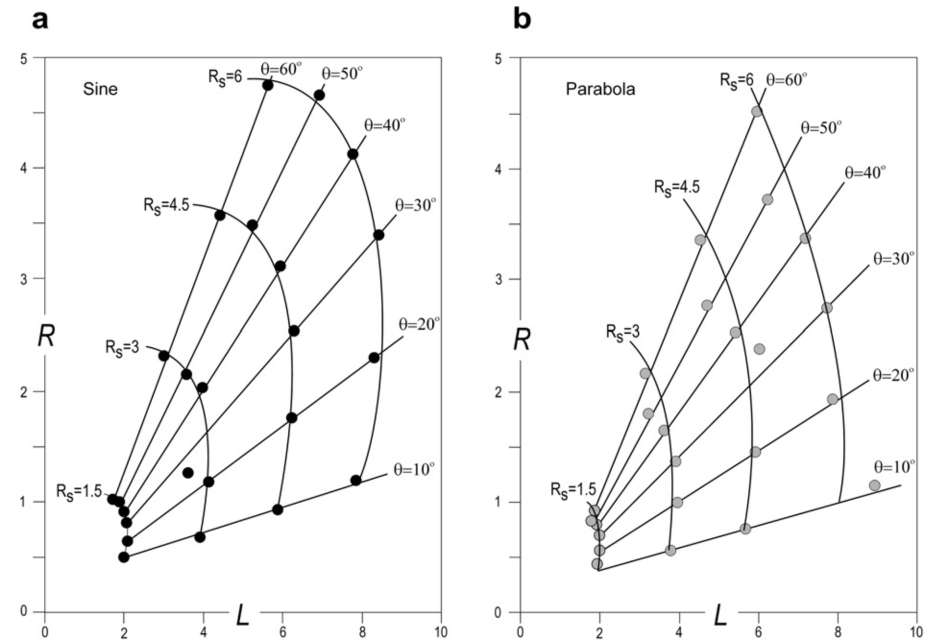

Fig. 6. (a) and (b) Point representation of 48 asymmetric fold shapes obtained from the sine curve and parabola, respectively. Straight lines are the trajectories of points joining equal angle of rotation (θ), whereas curves are the trajectories of points joining equal value of the strain ratio (R_s).

two-dimensional strain ratio $R_s = 1.5$, 3.0, 4.5 and 6.0, respectively, such that the maximum stretching direction is vertical in each distortion. Each combination of the rotation angle θ and the distortion R_s yields a characteristic asymmetric fold shape. A typical example of computer simulated asymmetric fold shape that is obtained by rotation of the sine curve by $\theta = 30°$ and the distortion of the rotated sine curve by the strain ratio $R_s = 1.5$ is given in Fig. 3c,d. The combinations of six rotations and four distortions, each for the sine curve and the parabola, produce 48 asymmetric fold shapes (examples in Figs. 4 and 5). We have tested the validity of the Bézier curve method on all the computer simulated folds.

Test of Bézier curve method on the 48 graphically simulated asymmetric folds shows that each fold shape plots as a unique point on the two-dimensional L–R frame (Fig. 6a,b). Two sets of trajectories can be drawn through these plots: (i) the best-fit straight lines through the points of equal rotation angle θ, and (ii) the best-fit curves through the points of equal distortion R_s (Fig. 6a,b). Each point on the L–R plot represents an asymmetric fold shape that can be simulated by the unique pair of R_s and θ values. In addition, the L or R values also reflect the tightness of the asymmetric fold shapes. To illustrate this, we have calculated the interlimb angles of different folds and have studied their variations with R and L. We have used the parametrically generated values of x and y from equations (3) and (4) to calculate the slope at the inflection points of the folds by numerical differentiation and, thus, the interlimb angle. The folds assume a progressively tighter shape with increase in the L and decrease in the R values (Figs. 7 a,b and 8 a,b).

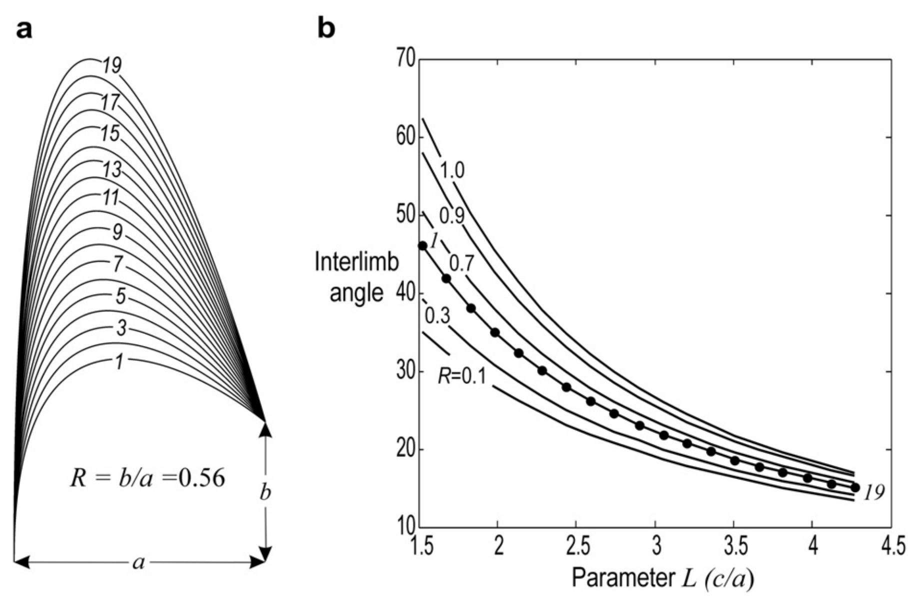

Fig. 7. (a) Computer graphics simulation of 19 asymmetric fold shapes by varying the parameter L at constant $R = 0.56$. (b) Relationship between L and the interlimb angle at $R = 0.1$, 0.3, 0.56, 0.7, 0.9 and 1.0. Folds 1–19 in (a) correspond to black dots in (b) The tightness of folds increases with progressive increase in L.

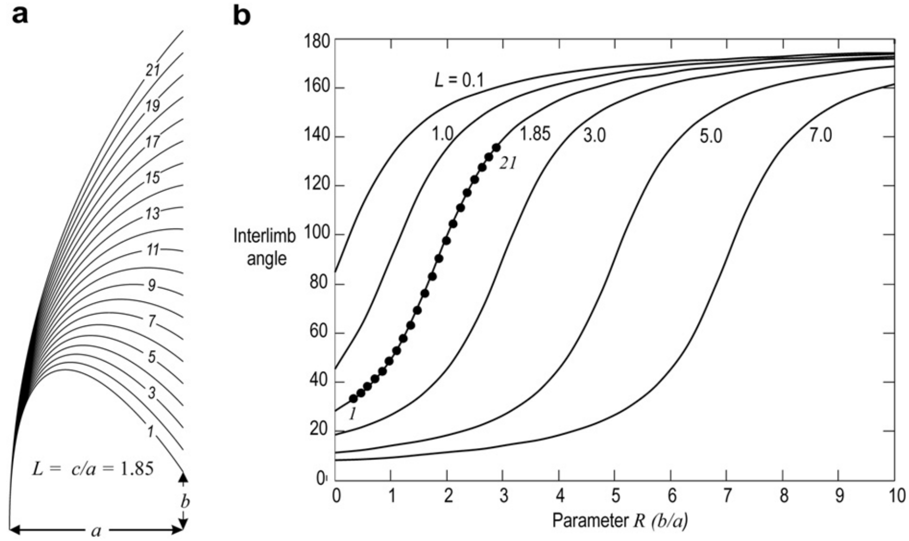

Fig. 8. (a) Computer graphics simulation of 21 asymmetric fold shapes by varying the parameter R at constant $L = 1.85$. (b) Relationship between R and the interlimb angle at $L = 0.1$, 1.0, 1.85, 3.0, 5.0 and 7.0. Folds become progressively open with progressive increase in R. Folds 1–21 in (a) correspond to black dots in (b).

4.2. Natural folds

We test the new method on a natural example that consists of seven asymmetric folds, A–G, in the psammite–pelite layers of the Moine Series, Mull, Scotland (Fig. 9a; taken from Fig. 15.15 in Ramsay and Huber, 1987, p. 318). We have particularly selected this example so that results of the fold shape analysis by the proposed method can be directly compared with the published results obtained from limb-by-limb analysis of these folds (Ramsay and Huber, 1987, p. 326; Fig. 5b in Srivastava and Lisle, 2004).

The results show two distinct shape clusters that consist of folds A–E and F–G, respectively (Fig. 9b). These results are consistent with those obtained by the limb-by-limb analysis of these folds by other methods (Ramsay and Huber, 1987, p. 326; Fig. 5b in Srivastava and Lisle, 2004). Besides, our results reveal that the R parameter is greater for cluster-*II* folds than that for cluster-*I* folds, whereas the difference in L parameter of the two clusters is rather small. These variations, when interpreted with reference to the relationships between the interlimb angle and the parameters L and R (Figs. 7 and 8), imply that the cluster-*I* folds are tighter and more sharp crested than relatively open cluster-*II* folds with broad hinge zones.

5. Merits and limitations of the new method

The Bézier curve method requires only two parameters, L and R, for tracing a complete asymmetric fold shape. It is noteworthy that parametric equations (3) and (4) produce different asymmetric fold shapes for different values L and/or R. The method, therefore, provides a mathematical basis for the unique point representation of a given asymmetric fold shape. An additional merit of the method is that the fold shape can be traced back by substitution of the coordinates of the representative point (L, R) in equations (3) and (4), respectively. For example, the substitution of the

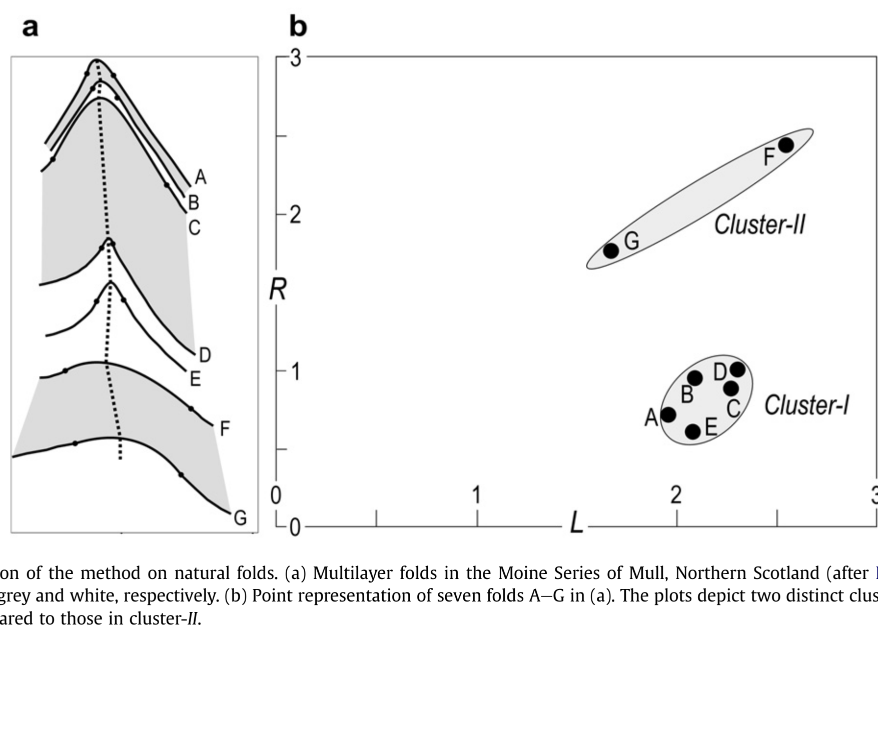

Fig. 9. An example of application of the method on natural folds. (a) Multilayer folds in the Moine Series of Mull, Northern Scotland (after Ramsay and Huber, 1987, p. 318). Psammitic and pelitic layers in grey and white, respectively. (b) Point representation of seven folds A–G in (a). The plots depict two distinct clusters, *I* and *II*. Folds in cluster-*I* are sharp crested and tighter compared to those in cluster-*II*.

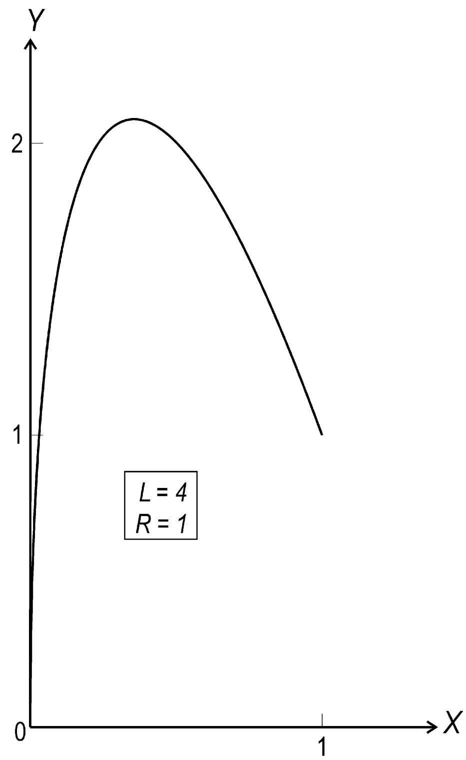

Fig. 10. An example for retrieving the fold shape from the point representation on the L–R plane. The fold shape has been obtained by substituting $L = 4$ and $R = 1$ in equations (3) and (4) in the text.

coordinates, say (4, 1) of a point in the equations (3) and (4) yields a unique fold shape (Fig. 10). The main limitation of the method is that it cannot be applied to some special fold shapes, e.g., double hinged folds, kink folds or elliptical folds due to difficulty in fitting Bézier curve on these shapes. Furthermore, the fit of Bézier curves on a few asymmetric folds is found to be imperfect in practice.

6. Conclusions

The Bézier curve method is a rapid and easy-to-use technique for the shape analysis of asymmetric folds. It shows that an asymmetric fold shape is a function of the two parameters, L and R that plot as a point on the two-dimensional Cartesian frame (Fig. 6a, b). Tests of numerous graphically simulated and natural fold examples validate the applicability of the method in the shape analysis and point representation of asymmetric folds.

Acknowledgements

Dipanwita Nandy initiated studies on point representation of folds at the Structural Geology laboratory, IIT Roorkee. We thank F. Bastida and an anonymous referee for the erudite reviews that helped improve the manuscript considerably. We are grateful to Richard Lisle for his advice and discussions, and Tom Blenkinsop for an efficient editorial handling. The work was supported by the Grants from Department of Science and Technology, Government of India.

References

Aller, J., Bastida, F., Toimil, N.C., Bobillo-Ares, N.C., 2004. The use of conic sections for the geometrical analysis of folded surface profiles. Tectonophysics 379, 239–254.
Bastida, F., Aller, J., Bobillo-Ares, N.C., 1999. Geometrical analysis of folded surfaces using simple functions. Journal of Structural Geology 21, 729–742.
Bastida, F., Aller, J., Bobillo-Ares, N.C., 2005. Fold geometry: a basis for their kinematical analysis. Earth Science Reviews 70, 129–164.
Bézier, P., 1966. Definition numérique des corbes et surfaces-I. Automatisme 12, 625–632.
Chapple, W.M., 1968. A mathematical theory of finite amplitude rock-folding. Geological Society of America Bulletin 79, 47–68.
Coelho, S., Passchier, C., Grasemann, B., 2005. Geometric description of flanking structures. Journal of Structural Geology 27, 597–606.
Davies, B.L., Robotham, A.J., Yarwood, A., 1986. Computer-aided Drawing and Design. Chapman and Hall, London, 328 pp.
De Paor, D.G., 1996. Bézier curves and geological design. In: De Paor, D.G. (Ed.), Structural Geology and Personal Computers. Pergamon Press, pp. 389–417.
Fleuty, M.J., 1964. The description of folds. Proceedings of the Geologists Association 75, 461–492.
Hudleston, P.J., 1973. Fold morphology and some geometrical implication of theories of fold development. Tectonophysics 16, 1–46.
Lisle, R.J., 1988. The superellipsoidal form of coarse clastic sediments particles. Mathematical Geology 20, 879–890.
Lisle, R.J., Martínez, F., Bobillo-Ares, N., Menéndez, O., Aller, J., Bastida, F., 2006. FOLD PROFILER: a MATLAB-based program for fold shape classification. Computer and Geosciences 32, 102–108.
Liu, C., Zhang, Y., Shi, B., 2009a. Geometric and kinematic modeling of detachment folds with growth strata based on Bézier curves. Journal of Structural Geology 31, 260–269.
Liu, C., Zhang, Y., Wang, Y., 2009b. Analysis of complete fold shape based on quadratic Bézier curves. Journal of Structural Geology 31, 575–581.
Loudon, T.V., 1964. Computer Analysis of Orientation Data in Structural Geology, Technical Report Geog. Branch off. Naval Research. ONR Task No. 389-135. Contribution No. 1228(26), No. 13, 130 pp.
Price, N.J., 1967. The initiation and development of asymmetrical buckle folds in non-metamorphosed competent sediments. Tectonophysics 4 (2), 173–201.
Price, N.J., Cosgrove, J.W., 1990. Analysis of Geological Structures. Cambridge University Press, Cambridge, 502 pp.
Ramsay, J.G., 1967. Folding and Fracturing of Rocks. McGraw-Hill, New York, 568 pp.
Ramsay, J.G., Huber, M.I., 1987. The Techniques of Modern Structural Geology. Folds and Fractures, vol. 2. Academic Press, New York, pp. 309–700.
Srivastava, D.C., Lisle, R.J., 2004. Rapid analysis of fold shape using Bézier curves. Journal of Structural Geology 26, 1553–1559.
Stabler, C.L., 1968. Simplified Fourier analysis of fold shapes. Tectonophysics 6, 343–350.
Stowe, C.W., 1988. Application of Fourier analysis for computer representation of fold profiles. Tectonophysics 156, 311–323.
Tripathi, A., Gairola, V.K., 1999. Fold symmetry – a quantitative description. Journal of Structural Geology 21, 719–727.
Twiss, R.J., 1988. Description and classification of folds in single surfaces. Journal of Structural Geology 10, 607–623.
Twiss, R.J., Moores, E.M., 1992. Structural Geology. W.H. Freeman and Company, New York, 532 pp.
Whitten, E.T.H., 1966. Structural Geology of Folded Rocks. Rand McNally and Company, Chicago, 678 pp.
Wojtal, S., Hughes, G., 2001. Using Bézier curves to analyze the shapes of folded surfaces. Abstracts with Program – Geological Society of America 33 (6), 26.